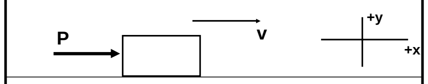
Friction

Push an object across a table at
a constant velocity.



Motion in the x direction: $a_x = 0$,

$\Sigma F_x = m\, a_x = 0$

There is a force P in the +x direction

There must be an additional force
in the -x direction.

The only contact of our object with another
object is the contact with the table.

The additional force must be caused by the
table and be parallel to its surface.

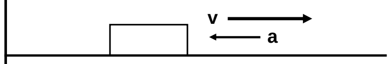
We name this force "kinetic friction", F_k.

Direction of Frictional Force

Can we always determine the direction
of the kinetic frictional force?

Try a simpler situation.

Block sliding on table with no one pushing.



The object always slows down

The kinetic frictional force can never
make an object go faster.

The kinetic frictional force is always
in the direction opposite to the
velocity of the object.

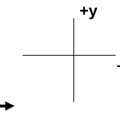
What determines the magnitude of this force?

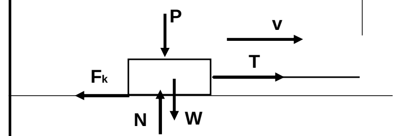
This is an experimental question which you
will work on in the lab.

Magnitude of Kinetic Friction

Pull a block across the table

Forces on the block:





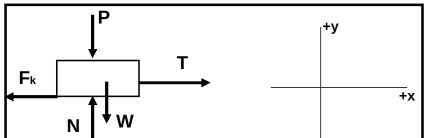
If you push down on an object, it takes a
larger force T to move the object at a
constant velocity.

P is in the y direction and cannot affect
motion in the x direction

Increasing P must increase a
force in the x direction

The kinetic frictional force increases.

How?



Motion in the y direction: $a_y = 0$, $\Sigma F_y = m\, a_y$

$\Sigma F_y = N + (-W) + (-P) = 0$

$N = W + P$

Increasing P, increases N

N is an interaction acting at the between the
table surface and the block.

Same place that F_k is acting.

It is reasonable that the kinetic frictional force
increases as the normal force increases.

Kinetic Friction

The simplest dependence is linear.

$F_k = k\, N$

F_k is in the direction opposite to v.

F_k is not in the direction of N

How something slides depends on the
material on the two surfaces that slide.

$k = \mu_k$ the coefficient of kinetic friction

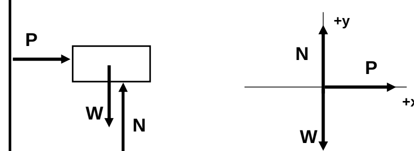
The theory of the behavior of kinetic friction:

$F_k = \mu_k F_N$ in the direction opposite to v.

Static Friction

An object is at rest relative to the
surface supporting it.

You push on it and it still doesn't move



If the object doesn't accelerate,
there must be another force in the - x direction.

A force between the surface on the object
and the surface of the table.

Static frictional force.

Direction and Magnitude

Direction

Opposite to P in direction parallel to surface

Magnitude

Varies

You push harder, the block still doesn't move

If you push hard enough, the block does accelerate.

The static frictional force has a maximum.

Maximum depends on:

Materials on the contact surface.

Coefficient of static friction, μ_s

How hard the surface pushes on the object

Normal force, N

Static Friction

Simple theory of static friction:

F_s (maximum) = μ_s N

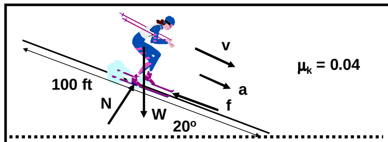
Direction is opposite to the acceleration that would occur without F_s

This theory is a simplified generalization of experience.

Check its applicability in your lab

Example Problem

While cross country skiing, you find you are on the top of a small hill. The hill side is a gentle slope at an angle of 20 degrees to the horizontal. You are new at this and are a bit afraid of going too fast since you are not good at stopping. You stop at the top of the hill and estimate that the hill side goes 100 feet before it gets to the bottom. How fast will you be going at the bottom of the hill? The coefficient of kinetic friction between your skis and the dry snow is 0.04.



Question:

What is the speed at the bottom of the hill?

Approach:

Get final speed from the acceleration.

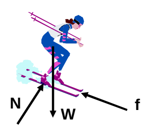
Get acceleration from the forces.

Perpendicular components of motion are independent.

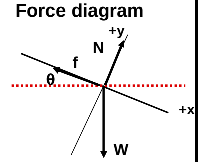
Interested in motion along the hill's slope.

Choose one axis of coordinate system along the slope of the hill. Call it x.

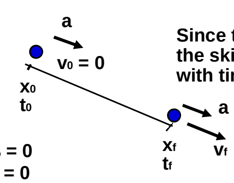
Free-body diagram



Force diagram



Motion diagram



Since the forces on the skier do not change with time, $a_o = a_f = a$

$x_o = 0$
$t_o = 0$
$v_o = 0$
a = ?

$\theta = 20°$
$\mu_k = 0.04$

x_f = 100 ft
t_f = ?
v_f = ?

Target : v_f

Relevant equations

for constant acceleration

$$\bar{a} = a = \frac{v_f - v_o}{t_f - t_o} = \frac{v_f}{t_f} \qquad x_f = \frac{1}{2} a t_f^2$$

$\Sigma F_x = m a_x$

$$W_x - f = ma$$

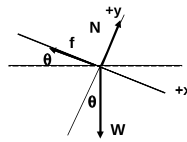
$\Sigma F_y = m a_y$

$$N - W_y = 0$$

$f = \mu N$

$W = m g$

components



$$\sin\theta = \frac{W_x}{W} \qquad \cos\theta = \frac{W_y}{W} \qquad \tan\theta = \frac{W_x}{W_y}$$

Plan: unknown

Find v_f — unknown: v_f

$a = \frac{v_f}{t_f}$ [1] — unknown: a , t_f

Find t_f

$x_f = \frac{1}{2} a t_f^2$ [2]

Find a

$mg\sin\theta - f = ma$ [3] — unknown: f, m

Find f

$f = \mu N$ [4] — unknown: N

Find N

$N - mg\cos\theta = 0$ [5]

6 unknowns but only 5 equations

Is there another equation for m??

Perhaps m cancels out.

Do the algebra and see if it does

$$N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

$f = \mu\, mg\cos\theta$

$$mg\sin\theta - \mu mg\cos\theta = ma$$

Yes! m cancels out

$$g\sin\theta - \mu g\cos\theta = a$$

$$x_f = \frac{1}{2}(g\sin\theta - \mu g\cos\theta)t_f^2$$

$$\sqrt{\frac{2x_f}{(g\sin\theta - \mu g\cos\theta)}} = t_f$$

$$g\sin\theta - \mu g\cos\theta = \frac{v_f}{\sqrt{\frac{2x_f}{(g\sin\theta - \mu g\cos\theta)}}}$$

$$(g\sin\theta - \mu g\cos\theta)\sqrt{\frac{2x_f}{(g\sin\theta - \mu g\cos\theta)}} = v_f$$

$$\sqrt{2x_f g(\sin\theta - \mu\cos\theta)} = v_f$$

Check units

$$\sqrt{[m]\left[\frac{m}{s^2}\right]} = v_f$$

$$\left[\frac{m}{s}\right] = v_f \qquad \text{ok}$$

$$\sqrt{2(100\text{ft})(32\frac{\text{ft}}{s^2})(\sin 20^o - 0.04\cos 20^o)} = v_f$$

v_f = 44 ft/s

Evaluate:

Units of velocity are correct, ft/s

A fast runner goes 100 m in 10 sec. which is 10 m/s

The skier goes about 14 m/s It's fast but possible

The is the speed of the skier at the bottom of the hill. The question is answered

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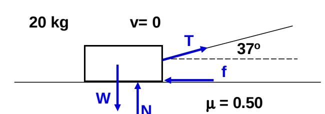
Review

- Picture with quantities and coordinate system
  - Situation, geometry, motion, forces.
  - Coordinate system x axis along acceleration
- Free-body and force diagrams
- Final velocity from constant acceleration kinematics
- Acceleration from forces in x direction
  - $\Sigma F_x = m\, a_x$
  - Add forces in x direction
    - W = mg
    - $F_k = \mu_k F_N$
- Normal force from motion in y direction
  - $a_y = 0$
  - $\Sigma F_y = m\, a_y$

EXAMPLE

A 20-kg crate sitting on a horizontal floor is attached to a rope that pulls 37 degrees above the horizontal. The coefficient of static friction between the crate and floor is 0.50. Determine the least rope tension that will cause the crate to start sliding.



Question:

What is force of rope on crate necessary to start the crate sliding?

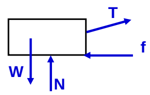
Approach:

Least rope tension to start sliding when its horizontal component equals the maximum static frictional force

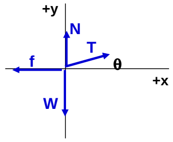
Components of forces
- horizontal
- vertical to get normal force

Consider crate just before it starts moving

Free-body diagram



Force diagram



Relevant equations

$\Sigma F_x = m a_x$

$T_x - f = 0$ (since $a_x = 0$)

$\Sigma F_Y = m a_Y$

$N - W - T_y = 0$ (since $a_y = 0$)

$W = mg$

$f \leq \mu_s N$

Components

$\sin\theta = \frac{T_y}{T}$ $\cos\theta = \frac{T_x}{T}$

Target : T

The plan:		unknown
Find T $\cos\theta = \frac{T_x}{T}$	1	T, T_x
Find T_x forces in the x direction $T_x - f = 0$; $f_{max} = T_x$	2	f_{max}
Find f_{max} $f_{max} = \mu_s N$	3	N
Find N Forces in the y direction: $N - mg - T_y = 0$	4	T_y
Find T_y $\sin\theta = \frac{T_y}{T}$	5	

5 unknowns, 5 equations
Can be solved.

$T \sin\theta = T_y$ [5]

$- T \sin\theta + mg = N$ [4]

$f_{max} = \mu_s (- T \sin\theta + mg)$ [3]

$\mu_s (- T \sin\theta + mg) = T_x$ [2]

$\cos\theta = \frac{\mu_s (- T \sin\theta + mg)}{T}$ [1]

$T \cos\theta = \mu_s (- T \sin\theta + m g)$

$T \cos\theta + \mu_s T \sin\theta = \mu_s m g$

$T (\cos\theta + \mu_s \sin\theta) = \mu_s m g$

$$T = \frac{\mu_s m g}{\cos\theta + \mu_s \sin\theta}$$

$$T = \frac{\mu_s mg}{(\cos\theta + \mu_s \sin\theta)}$$

$$T = \frac{(.50)(20\ kg)(9.8\ m/s^2)}{[(\cos 37°) + (.50)(\sin 37°)]}$$

$$T = \frac{98\ (kg\ m/s^2)}{(1.1)} = 89\ kg\ m/s^2$$

T = 89 N

Evaluate:

Correct units: force is in Newtons

Force required to lift box is

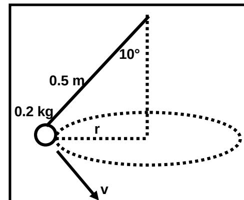
$(20\ kg)(10\ m/s^2) = 200\ N$

89 N is less than that which is reasonable

The least rope tension for the box to slide is when the static frictional force is at its maximum. The question is answered.

Example - Chap 6 -55

A conical pendulum is an object which is free to move at the end of a very light string with the other end of the string fixed. Given the proper push, the object can swing in a horizontal circle such that the string is always at the same angle to the vertical and the object maintains the same distance from the fixed point. If the mass of the object is 0.20 kg, the length of the string is 0.50 m, and the angle from the vertical is 10°, what is the speed of the object?



What is the speed of the object?

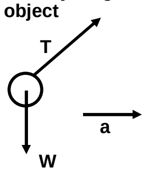
Kinematics for circular motion relates the speed of an object to its acceleration.

Can get acceleration of the object from the forces on the object.

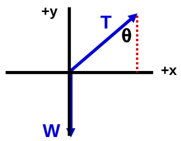
Choose one coordinate along the radius of the circle.

Ignore air resistance and any friction of the string rubbing against the support.

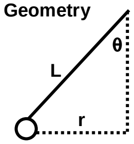
Free body diagram of object



Force diagram of object



Geometry



L = 0.50 m

m = 0.20 kg

$\theta = 10°$

$\sin\theta = \frac{r}{L}$

$\sin\theta = \frac{T_x}{T}$

$\cos\theta = \frac{T_y}{T}$

$\tan\theta = \frac{T_x}{T_y}$

Forces in x direction: $T_x = ma$

Forces in y direction: $T_y - W = ma_y = 0$

$W = mg$ $\qquad a = \frac{v^2}{r}$

Target: v

	unknowns
Find v	v
$a = \frac{v^2}{r}$ **1**	a, r
Find r	
$\sin\theta = \frac{r}{L}$ **2**	
Find a	
$T_x = ma$ **3**	T_x
Find T_x	
$\tan\theta = \frac{T_x}{T_y}$ **4**	T_y
Find T_y	
$T_y - mg = 0$ **5**	

5 unknowns, 5 equations ok

$T_y = mg$

$\frac{T_x}{\tan\theta} = mg$

$T_x = mg\tan\theta$

$mg\tan\theta = ma$

$g\tan\theta = a$

$g\tan\theta = \frac{v^2}{r}$

$L\sin\theta = r$

$g\tan\theta = \frac{v^2}{L\sin\theta}$

$L\,g\sin\theta\tan\theta = v^2$

$\sin\theta\sqrt{\frac{Lg}{\cos\theta}} = v$

Check Units:

$$\sin\theta\sqrt{\frac{Lg}{\cos\theta}} = v$$

$$\sqrt{[m]\left[\frac{m}{s^2}\right]} = \frac{m}{s} \qquad \text{ok}$$

$$\sin 10^o\sqrt{\frac{(0.50m)\left(9.8\frac{m}{s^2}\right)}{\cos 10^o}} = v = 0.39\frac{m}{s}$$

m/s are correct units for a speed.

The question was answered by finding the speed of the object.

Circle has a radius of $L\sin\theta =$ r = 0.09 m

Circumference = 0.55m

Time to go around circle =Period = 1.4 second

A reasonable time

0.4 m/s is not an unreasonable speed for an object swinging at the end of a string.